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AD No. 241 531

Adaptive Switching Circuits

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by

B. Widrow

M. E. Hoff

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Technical Report No. 1553-1

June 30, 1960

Prepared under Office of Naval Research Contract
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Jointly supported by the U.S. Army Signal Corps, the
U.S. Air Force, and the U.S. Navy (Office of Naval Research)

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SOLID-STATE ELECTRONICS LABORATORY
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ADAPTIVE SWITCHING CIRCUITS

by

B. Widrow

M. E. Huff

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**Solid-State Electronics Laboratory
Stanford Electronics Laboratories
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Stanford, California**

SUMMARY

Adaptive or "learning" systems can automatically modify their own structures to optimize performance based on past experiences. The system designer "teaches" by showing the system examples of input signals or patterns and simultaneously what he would like the output to be for each input. The system in turn organizes itself to comply as well as possible with the wishes of the designer.

An adaptive pattern classification machine (called "Adaline", for adaptive linear) has been devised to illustrate adaptive behavior and artificial learning. During a training phase, crude geometric patterns are fed to the machine by setting the toggle switches in a 4×4 input array. Setting another toggle switch tells the machine whether the desired output for the particular input pattern is +1 or -1. All input patterns are classified into two categories. The system learns a little from each pattern and accordingly experiences a design change. After training, the machine can be used to classify the original patterns and noisy (distorted) versions of these patterns.

A statistical theory has been developed which relates the competence of the classifier to the amount of experience had (number of patterns "seen" in adapting). Imperfect system adjustment results from small-sample-size experience. The misadjustment, a dimensionless quantitative measure of the quality of adaption, is defined as the ratio of the increase in probability of error of a system adapted to a small number of patterns to the probability of error of a "best-adapted" system (adapted to an arbitrarily large number of patterns). Treating the classifier as a roughly quantized sampled-data system, a statistical theory of adaption developed for adaptive sampled-data systems has been utilized to derive a formula for misadjustment,

$$M = \frac{n+1}{N}$$

The number of input lines is $(n + 1)$, and the number of patterns seen in adapting is N. This formula leads to a basic "rule of thumb" for adaptive classifiers: The number of patterns required to train an adaptive classifier is equal to several times the number of bits per pattern. This

rule applies without regard to patterns and noise characteristics. Experimental evidence is presented.

The pattern classifier is actually an adaptive switching circuit having a set of binary inputs and a binary output. The signal on each input line is either +1 or -1 according to the setting of the individual pattern switch. The sixteen input signals are linearly combined and then quantized. The weights (which could be positive or negative) are determined by an array of potentiometer settings.

Iterative gradient methods are used during the training phase to find the potentiometer settings that minimize the number of classification errors. A simple procedure has been devised which does not require actual measurement of gradient, and which guarantees convergence and permits control of rate of convergence. Adaline can usually adapt after seeing ten to twenty patterns and can easily distinguish a dozen different basic patterns.

As a generic form of switching functions, Adaline is not completely general. All-possible-potentiometer-settings allows the realization of the "linearly separated truth functions", a subclass of all switching functions. Although this subclass is restricted, it is a useful class, and, most important, it is a searchable class (the best within the class can be found without trying all possibilities). Networks of Adalines overcome this restriction and are far more general, yet present adaption problems of no greater difficulty than those of single Adalines.

At present the purely mechanical adaption process is accomplished by manual potentiometer-setting. A means of automating this is being developed which makes use of multi-aperture ferromagnetic devices. Solid-state adaptive logical elements will result that should ultimately be suitable to be microminiaturized. Networks of such elements would be very effective in pattern recognition systems, information storage and retrieval-by-classification systems, and self-repairing logical and computing systems.

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I. INTRODUCTION

The modern science of switching theory began with work by Shannon¹ in 1938. The field has developed rapidly since then, and at present a wealth of literature exists² concerning the analysis and synthesis of logical networks which might range from simple interlock systems to telephone switching systems to large-scale digital computing systems.

A key idea in switching theory is that the performance requirements of any logical system can be completely specified by a boolean function expressing output conditions in terms of input conditions, and that the algebraic symbols in the boolean function are readily identifiable with simple storage and "and" - "or" elements. The problem of simplification of networks for most economic realization is reduced to a problem of algebraic simplification of boolean functions, a task which is more easily accomplished by human designers than reduction-by-inspection of logical networks themselves.

An example illustrating the use of switching theory is that of the design of an interlock system for the control of traffic in a railroad switch yard. The first step is the preparation of a "truth table", an exhaustive listing of all input possibilities (the positions of all incoming and outgoing trains), and what the desired system output should be (what the desired control signals should be) for each input situation. The next step is the construction of the boolean function, and the following steps are algebraic reduction and design of the logical control system.

The design of a traffic control system is an example wherein the truth table must be followed precisely and reliably. Errors would be destructive.

The design of the arithmetic element of a digital computer is another example wherein the truth table must be followed precisely. There are other situations in which some errors are inevitable, however, and here errors are usually costly but not catastrophic. These situations call for statistically optimum switching circuits. A common performance objective is the minimization of the average number of errors. An example is that of prediction of the next bit in a correlated stochastic

binary number sequence. The predictor output is to be a logical combination of a finite number of previous input sequence bits. An optimum system is a sequential switching circuit that predicts with a minimum number of errors.

Suppose that a record of the binary sequence is printed on tape and cut up into pieces (with indication of the positive direction of time preserved), say 25 bits long. Place all pieces where the most recent event is ONE in one pile and the remainder in another pile. Delete the most recent bit on each piece of tape. If the statistical scheme could be discovered by which the pieces of tape are classified, this would lead to a prediction scheme. It is apparent that prediction is a certain kind of classification.

Assuming statistical regularity, a reasonable way to proceed might be to form a truth table, and let the data from each piece of tape be an entry in the table. It might be expected that with the data of 100 pieces of tape, a fairly good predictor could be developed. The truth table would have only 100 entries however, out of a total of 2^{24} . The "best" way to fill in the remainder of the truth table depends upon the nature of the sequence statistics and the error cost criteria. Filling in the table is a difficult and a crucial part of the problem. Even if the truth table were filled in, however, the designer would have the difficult task of realizing a logical network to satisfy a truth table with 2^{24} entries.

An approach to such problems is taken in this paper which does not require an explicit use of the truth table. The design objective is the minimization of the average number of errors, rather than a minimization of the number of logical components used. The nature of the logical elements is quite unconventional. The system design procedure is adaptive, and is based upon an iterative search process. Performance feedback is used to achieve automatic system synthesis, i.e., the selection of the "best" system from a restricted but useful class of possibilities. The designer "trains" the system to give the correct responses by "showing" it examples of inputs and respective desired outputs. The more examples "seen", the better is the system performance. System competence will be directly and quantitatively related to amount of experience.

II. A NEURON ELEMENT

In Fig. 1, a combinatorial logical circuit is shown which is a typical element in the adaptive switching circuits to be considered. This element bears some resemblance to a "neuron" model introduced by von Neuman³, whence the name.

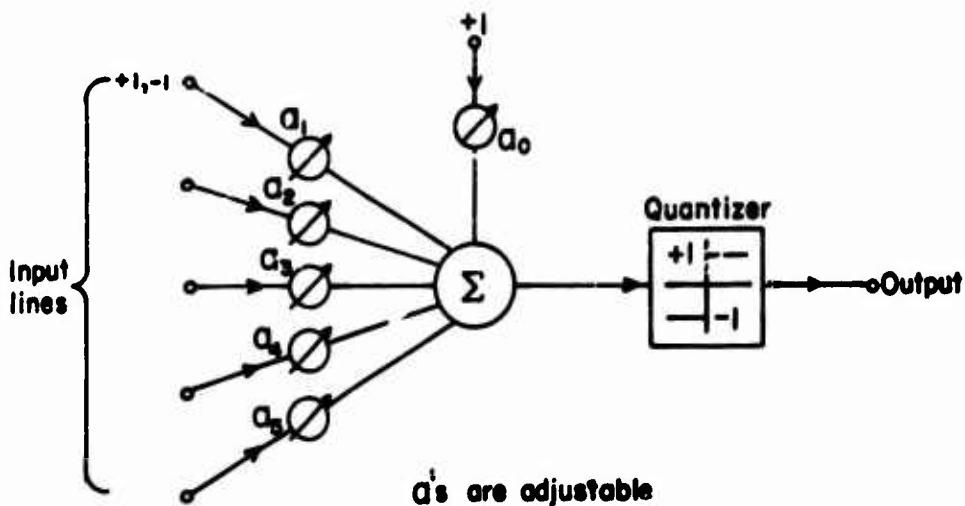


FIG. 1...AN ADJUSTABLE NEURON.

The binary input signals on the individual lines have values of +1 or -1, rather than the usual values of 1 or 0. Within the neuron, a linear combination of the input signals is formed. The weights are the gains a_1, a_2, \dots , which could have both positive and negative values. The output signal is +1 if this weighted sum is greater than a certain threshold, and -1 otherwise. The threshold level is determined by the setting of a_0 , whose input is permanently connected to a +1 source. Varying a_0 varies a constant added to the linear combination of input signals.

For fixed gain settings, each of the 2^5 possible input combinations would cause either a +1 or -1 output. Thus, all possible inputs are classified into two categories. The input-output relationship is determined by choice of the gains a_0, \dots, a_5 . In the adaptive neuron, these gains are set during the "training" procedure.

In general, there are 2^{2^5} different input-output relationships or truth functions by which the five input variables can be mapped into the single output variable. Only a subset of these, the linearly separated truth functions⁴, can be realized by all possible choices of the gains of the neuron of Fig. 1. Although this subset is not all-inclusive*, it is a useful subset, and it is "searchable", i.e., the "best" function in many practical cases can be found iteratively without trying all functions within the subset.

Application of this neuron in adaptive pattern classifiers was first made by Mattson.^{5,6} He has shown that complete generality in choice of switching function could be had by combining these neurons. He devised an iterative digital computer routine for finding the best set of a's for the classification of noisy geometric patterns. An iterative procedure having similar objectives has been devised by these authors and is described in the next section. The latter procedure is quite simple to implement, and can be analyzed by statistical methods that have already been developed for the analysis of adaptive sampled data systems.

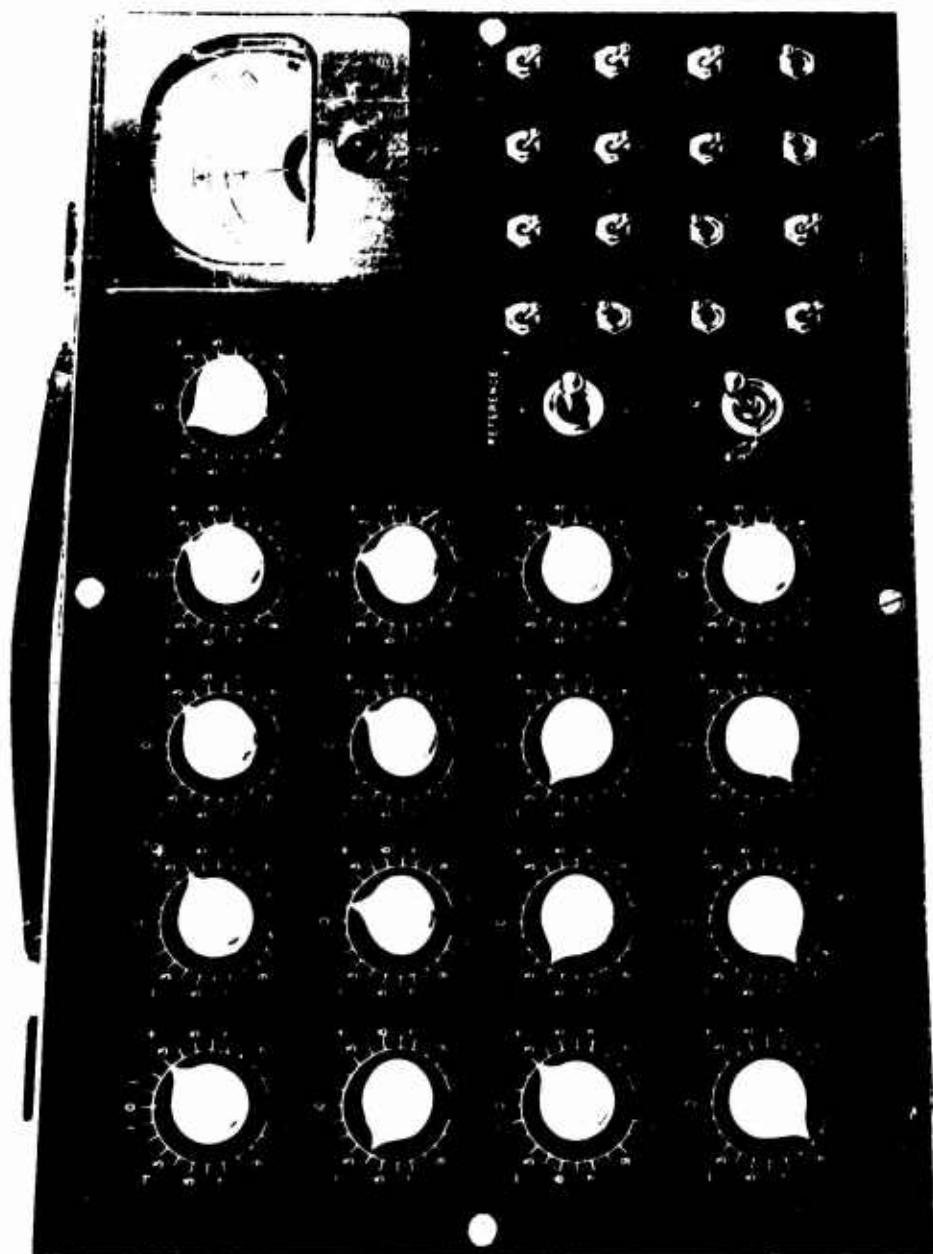
III. AN ADAPTIVE PATTERN CLASSIFIER

An adaptive pattern classification machine (called "Adaline", for adaptive linear) has been constructed for the purpose of illustrating adaptive behavior and artificial learning. A photograph of this machine, which is about the size of a lunch pail, is shown in Fig. 2.

During a training phase, crude geometric patterns are fed to the machine by setting the toggle switches in the 4x4 input switch array. Setting another toggle switch (the reference switch) tells the machine whether the desired output for the particular input pattern is +1 or -1. The system learns a little from each pattern and accordingly experiences a design change. The machine's total experience is stored in the values of the weights $a_0 \dots a_{16}$. The machine can be trained on undistorted noise-free patterns by repeating them over and over until the iterative search process converges, or it can be trained on a sequence of noisy

* It becomes a vanishingly small fraction of all possible switching functions as the number of inputs gets large.

FIG. 2. - ADALINE.



patterns on a one-pass basis such that the iterative process converges statistically. Combinations of these methods can be accommodated simultaneously. After training, the machine can be used to classify the original patterns and noisy or distorted versions of these patterns.

A block schematic of Adaline is shown in Fig. 3. In the actual machine, the quantizer is not built in as a device but is accomplished by the operator in viewing the output meter. Different quantizers (2-level, 3-level, 4-level) are realized by using the appropriate meter scales (see Fig. 2). Adaline can be used to classify patterns into several categories by using multi-level quantizers and by following exactly the same adapting procedure.

The following is a description of the iterative searching routine. A pattern is fed to the machine, and the reference switch is set to correspond to the desired output. The error (see Fig. 3) is then read (by switching the reference switch; the error voltage appears on the meter, rather than the neuron output voltage). All gains including the level are to be changed by the same absolute magnitude, such that the error is brought to zero. This is accomplished by changing each gain

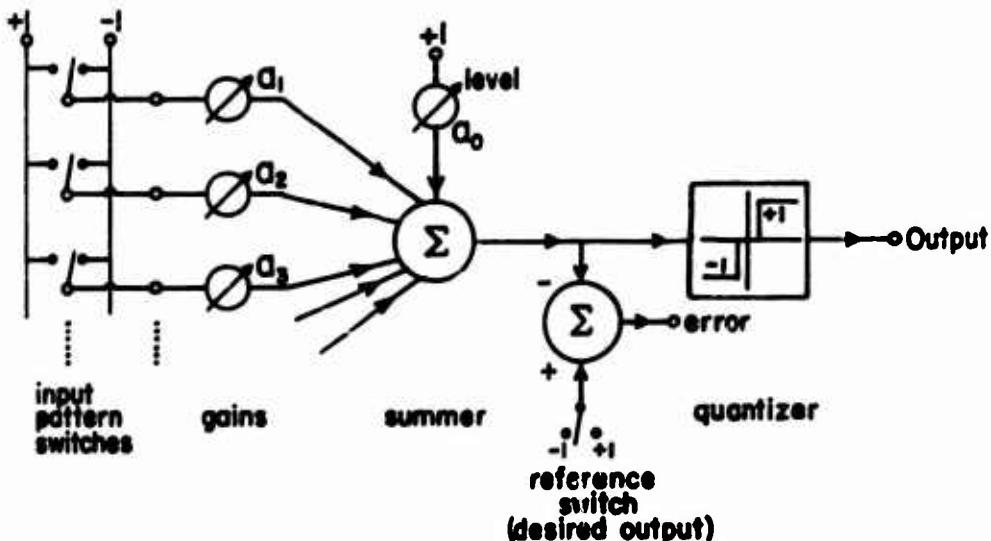


FIG. 3.--SCHEMATIC OF ADALINE.

(which could be positive or negative) in the direction which will diminish the error by an amount which reduces the error magnitude by 1/17. The 17 gains may be changed in any sequence, and after all changes are made, the error for the present input pattern is zero. Switching the reference back, the meter reads exactly the desired output. The next pattern, and its desired output, is presented and the error is read. The same adjustment routine is followed and the error is brought to zero. If the first pattern were reapplied at this point, the error would be small but not necessarily zero. More patterns are inserted in like manner. Convergence is indicated by small errors (before adaptation), with small fluctuations about a stable root mean-square value. The iterative routine is purely mechanical, and requires no thought on the part of the operator. Electronic automation of this procedure will be discussed below.

The results of a typical adaption on six noiseless patterns is given in Figs. 4 and 5. The patterns were selected in a random sequence, and were classified into 3 categories. Each T was to be mapped to +60 on the meter dial, each G to 0, and each F to -60. As a measure of performance, after each adaptation, all six patterns were read in (without adaptation) and six errors were read. The sum of their squares denoted by Σe^2 was computed and plotted. Figure 5 shows the learning curve for the case in which all gains were initially zero.

IV. STATISTICAL THEORY OF ADAPTION FOR SAMPLED-DATA SYSTEMS

This section is a summary of the portions of Widrow's statistical theory of adaption for sampled-data systems^{7,8}, that is useful in the analysis of adaptive switching circuits.

Consider the general linear sampled-data system formed of a tapped delay line, shown in Fig. 6. This system is intended to be a statistical predictor. The present output sample $g(m)$ is a linear combination of present and past input samples, and is intended to approximate as closely as possible the next input sample $f(m+1)$. The constants in this linear combination are h_1, h_2, h_3 , etc., the predictor impulse-response samples, or the gains associated with the delay-line taps. Their choice constitutes

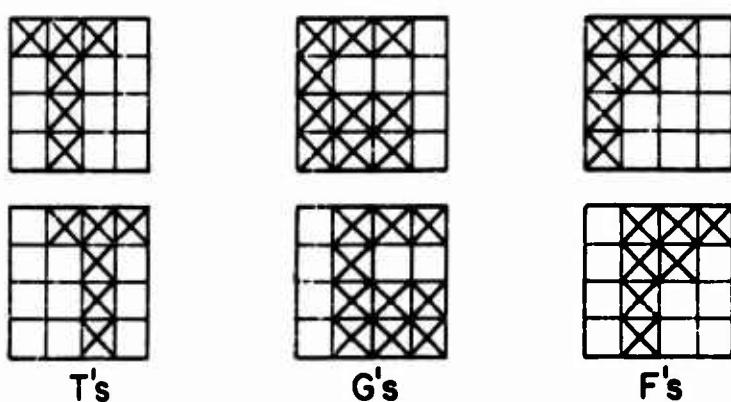


FIG. 4---PATTERNS FOR CLASSIFICATION EXPERIMENT.

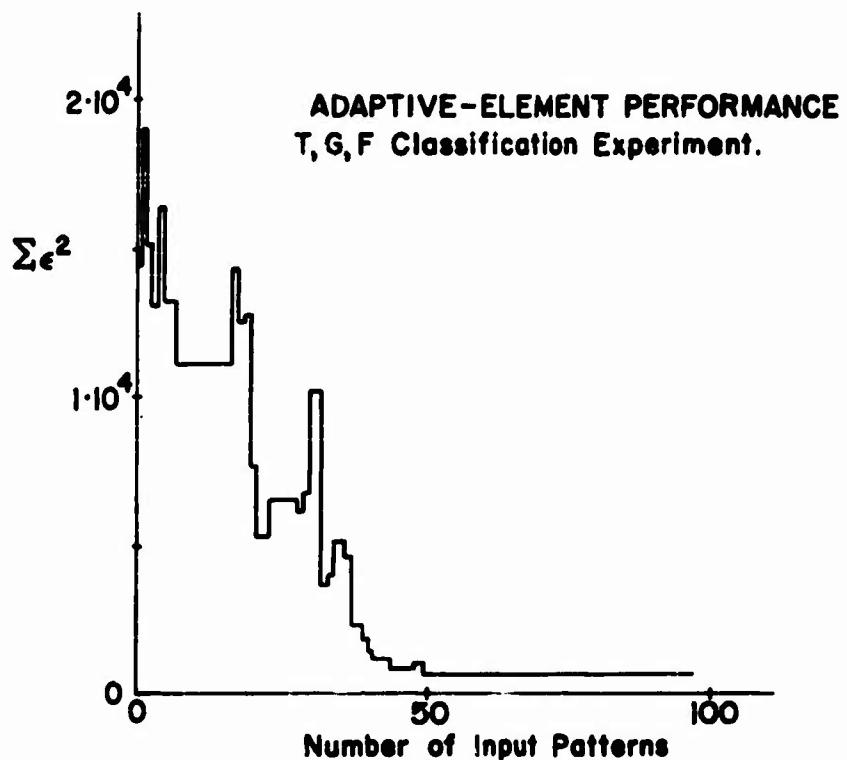


FIG. 5---ADAPTIVE-ELEMENT PERFORMANCE CURVE.

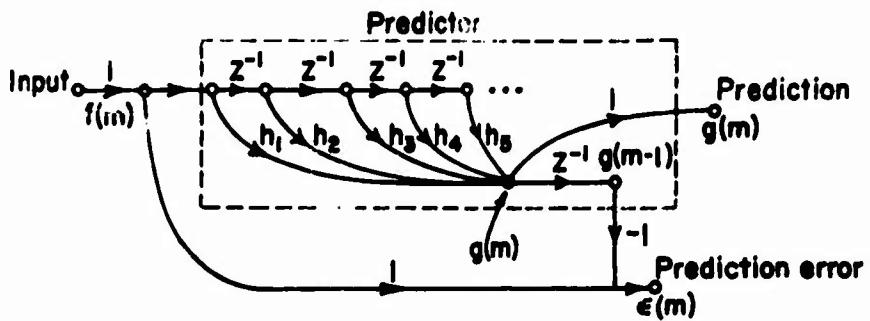


FIG. 6... AN ADJUSTABLE SAMPLED-DATA PREDICTOR.

the adjustable part of the predictor design. They may be adjusted in the following manner. Apply a mean square reading meter to $\epsilon(m)$, the difference between the present input and the delayed prediction. This meter will measure mean square error in prediction. Adjust h_1, h_2, h_3, \dots , until the meter reading is minimized.

The problem of adjusting the h 's is not trivial, because their effects upon performance interact. Suppose that the predictor has only two impulses on its impulse response, h_1 and h_2 . The mean square error for any setting of h_1 and h_2 can be readily derived:

$$\begin{aligned}\epsilon(m) &= f(m) - h_1 f(m-1) - h_2 f(m-2) \\ \overline{\epsilon^2}(m) &= \phi_{ff}(0)h_1^2 + \phi_{ff}(0)h_2^2 - 2\phi_{ff}(1)h_1 - 2\phi_{ff}(2)h_2 \\ &\quad + 2\phi_{ff}(1)h_1 h_2 + \phi_{ff}(0)\end{aligned}\quad (1)$$

The discrete autocorrelation function of the input is $\phi_{ff}(j)$.

The mean square error given by equations (1) is what the mean square meter would read if it were to average over very large sample size. The mean square error is a parabolic function of the predictor adjustments h_0 and h_1 , and, in general, can easily be shown to be a quadratic function of such adjustments, regardless of how many there are.

The optimum n-impulse predictor can be derived analytically by setting the partial derivatives of $\overline{\epsilon^2}$ of equation (1) equal to zero. This is the discrete analogue of Wiener's optimization⁹ of continuous filters.

Finding the optimum system experimentally is the same as finding a minimum of a paraboloid in n dimensions. This could be done manually by having a human operator read the meter and set the adjustment, or it could be done automatically by making use of any one of several iterative gradient methods for surface-searching, as devised by numerical analysts. When either of these schemes is employed, an adaptive system results that consists essentially of a "worker" and a "boss". The worker in this case predicts, whereas the boss has the job of adjusting the worker.

Figure 7 is a block-diagram representation of such a basic adaptive unit. The boss continually seeks a better worker by trial and error experimentation with the structure of the worker. Adaption is a multi-dimensional performance feedback process. The "error" signal in the feedback control sense is the gradient of mean square error with respect to adjustment.

Many of the commonly used gradient methods search surfaces for stationary points by making changes in the independent variables (starting with an initial guess) in proportion to measured partial derivatives to obtain the next guess, and so forth. These methods give rise to geometric (exponential) decays in the independent variables as they approach a stationary point for second-degree or quadratic surfaces. One-dimensional surface-searching is illustrated in Fig. 8.

The surface being explored in Fig. 8 is given by Eq. (2). The first and second derivatives are given by Eq. (3) and (4).

$$y = a(x - b)^2 + c \quad (2)$$

$$\frac{dy}{dx} = 2a(x - b) \quad (3)$$

$$\frac{d^2y}{dx^2} = 2a \quad (4)$$

A sampled-data feedback model of the iterative process is shown in Fig. 8b. Each time a guess in x is to be made, the derivative is measured physically whereas in the model it is formed as a quantity proportional to x (according to Eq. 3).

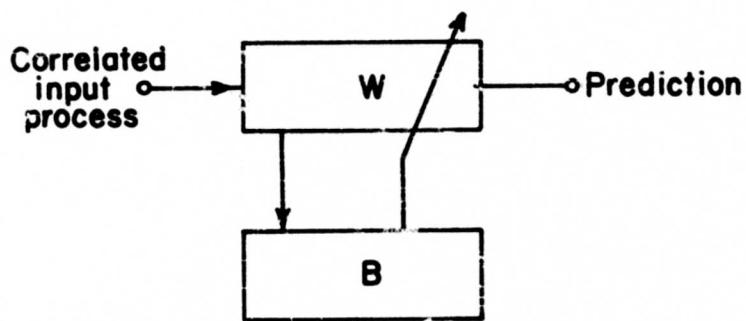


FIG. 7.--AN ADAPTIVE PREDICTOR.

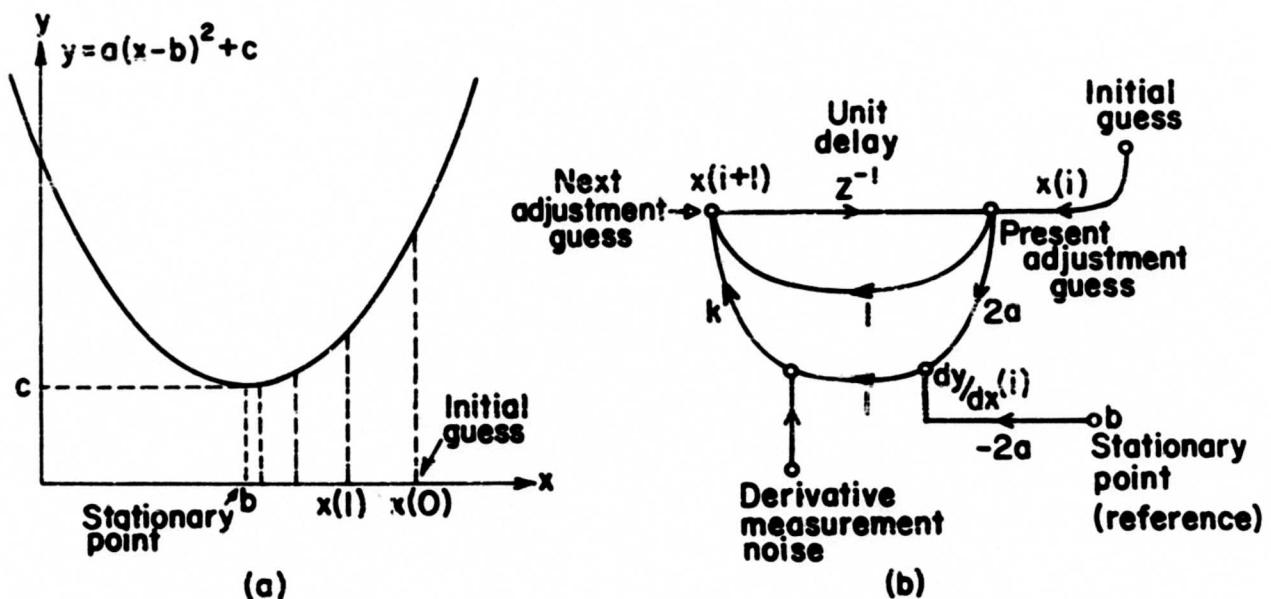


FIG. 8.--ONE-DIMENSIONAL SURFACE SEARCHING.

The numerical sequence at the point $x(1)$ begins with the initial guess and proceeds as a sampled transient that relaxes geometrically toward the stationary point, exactly like the sequence of guesses in the surface exploration.

The flow-graph can be reduced, and the transfer function from any point to any other point can thus be found. The resulting characteristic equation is

$$-(2ak + 1)z^{-1} + 1 = 0$$

The iterative process is stable when

$$0 > k > -\frac{1}{2a} \quad (5)$$

In order to choose the "loop gain" k to get a specific transient decay rate, one would have to measure the second derivative $(2a)$ at some point on the curve. Transients decay completely in one step when

$$k = -\frac{1}{2a}$$

Derivatives are measured in the actual adaptive system by varying the h 's by fixed increments and subtracting measured values of mean square error based on a sample size of N samples. "Noise" in the measurements of the mean square error surface due to small sample size cause noisy derivative measurements. These noises enter the adaption process, as indicated in Fig. 8o, and cause noisy system adjustments. The larger the sample size taken per derivative measurement, the less is the noise. The slower the adaptation, the more precise it is. The faster the adaptation, the more noisy (and poor) are the adjustments.

Consider that the adaptive model has only a single adjustment. A plot of mean square error versus h_1 for this simplest system would be a parabola, analogous to the parabola of Fig. 8. Noise in the system adjustment causes loss in steady-state performance. It is useful to define a dimensionless parameter M the "misadjustment", as the ratio of the mean increase in mean square error to the minimum mean square error. It is a measure of how the system performs on the average, after adapting transients have died out, compared with the fixed optimum system. With regard to the curve of Fig. 8,

$$M = \frac{\bar{y} - c}{c} \quad (6)$$

Variance in x about the optimum value causes the average of y to be greater than the minimum value c . The increase in \bar{y} equals the variance in x multiplied by a , as can be seen from Eq. (2).

More detailed derivations of misadjustment formulas covering several different methods of surface searching and derivative measurement are presented in Refs. 7 and 8. The particular formulas which can be applied to the analysis of adaptive switching circuits are the following.

When derivatives are measured by data repeating, i.e., when the same system input data is applied for both the N "forward" and the N "backward" measurements of mean square error, the misadjustment is given by

$$M = \frac{1}{2(N\tau)} \quad (7)$$

τ is the time constant of the iterative process of Fig. 8, and is equal to $-(1/2ak)$. A unit time constant means that the adjustment error decreases by a factor $1/e$ per iteration cycle. Equation (7) is conservative, and appreciably so only for small values of τ , less than 1. In the limiting case of one-step adaption, $\tau = 0$ and the appropriate misadjustment formula is

$$M = \frac{1}{N} \quad (8)$$

In deriving Formulas (7) and (8), it has been assumed that the error samples are gaussian distributed, with zero mean, and are uncorrelated. It can be shown that these results are highly insensitive to this distribution density shape, and are appreciably affected by correlation only when it exceeds 0.8.

It is interesting to note that the quality of adaption depends only on the number of samples "seen" by the system in adapting. When Eq. (7) applies, the $(N\tau)$ product determines the misadjustment. This product is equal to the number of samples seen per time constant of adaptation. If it may be considered that transients die out within two time constants, then the misadjustment equals the reciprocal of the number of samples that elapse in adapting to a step change in process. This statement is obviously the case when Eq. (8) applies.

The expressions (7) and (8) are based on the supposition that fresh data is brought in for each cycle of iteration. If the system adapts on a fixed body of N error samples, either by adapting with the one-step procedure and stopping, or by repeating the same data from iteration cycle to iteration cycle for several time constants and then stopping, the adjustment is given by Formula (8).

When there are m interacting adjustments instead of just one, Expressions (7) and (8) may be generalized by multiplication by m . Multi-dimensional one-step surface searching may be accomplished by Newton's method. Multi-step searching may be conveniently achieved by means of the method of steepest descent (making changes in adjustment in the direction of the surface gradient and in proportion to its magnitude) or by the Southwell relaxation method (cyclic adjustment for minima, one coordinate at a time).

V. STATISTICAL THEORY OF ADAPTION FOR THE ADAPTIVE NEURON ELEMENT

The error signal measured and used in adaption of the neuron of Fig. 1 is the difference between the desired output and the sum before quantization. This error is indicated by ϵ in Fig. 9. The actual neuron error, indicated by ϵ_n in Fig. 9, is the difference between the neuron output and the desired output.

The objective of adaption is the following. Given a collection of input patterns and the associated desired outputs, find the best set of weights a_0, a_1, \dots, a_m to minimize the mean square of the neuron error, $\overline{\epsilon_n^2}$. Individual neuron errors could only have the values of +2, 0, and -2 with a two-level quantizer. Minimization of $\overline{\epsilon_n^2}$ is therefore equivalent to minimizing the average number of neuron errors.

The simple adaption procedure described in this paper minimizes $\overline{\epsilon^2}$ rather than $\overline{\epsilon_n^2}$. The measured error ϵ has zero mean (a consequence of the minimization of $\overline{\epsilon^2}$) and will be assumed to be gaussian-distributed. By making use of certain geometric arguments or by using a statistical theory of amplitude quantization,¹⁰ it can be shown that $\overline{\epsilon_n^2}$ is a monotonic function of $\overline{\epsilon^2}$, and that minimization of $\overline{\epsilon^2}$ is equivalent to minimization of $\overline{\epsilon_n^2}$.

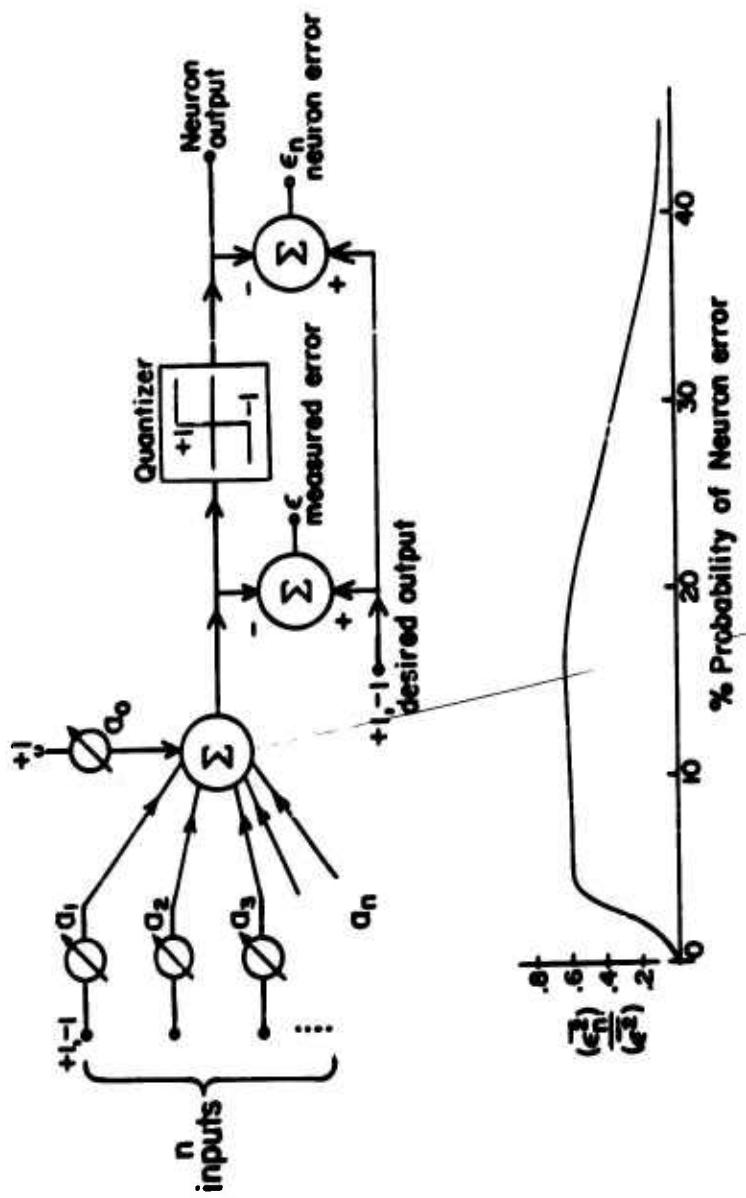


FIG. 9.- RELATIONS BETWEEN NEURON ERRORS AND MEASURED ERRORS.

and to minimization of the probability of neuron error.* The ratio of these mean squares has been calculated and is plotted in Fig. 9 as a function of the neuron error probability (which is $\epsilon^2/n^2/4$).

Given any collection of input patterns and the associated desired outputs, the measured mean square error $\bar{\epsilon}^2$ must be a precisely parabolic function of the gain settings, a_0, \dots, a_n . Let the k th pattern be indicated as the vector $S(k) = s_1(k), s_2(k), \dots, s_n(k)$. The s 's have values of +1 or -1, and represent the n input components numbered in a fixed manner. The k th error is

$$\epsilon(k) = d(k) - a_0 - a_1 s_1(k) - a_2 s_2(k) - \dots - a_n s_n(k) \quad (9)$$

For simplicity, let the neuron have only two input lines and a level control. The square of the error is accordingly

$$\begin{aligned} \epsilon^2(k) &= d^2(k) + a_0^2 + s_1^2(k)a_1^2 + s_2^2(k)a_2^2 \\ &\quad - 2d(k)a_0 - 2d(k)s_1(k)a_1 - 2d(k)s_2(k)a_2 \\ &\quad + 2s_1(k)a_0 a_1 + 2s_2(k)a_0 a_2 + 2s_1(k)s_2(k)a_1 a_2 \end{aligned} \quad (10)$$

The mean square error averaged over k is

$$\begin{aligned} \bar{\epsilon}^2 &= a_0^2 + \theta(s_1, s_1)a_1^2 + \theta(s_2, s_2)a_2^2 - \bar{d}a_0 \\ &\quad - 2\theta(d, s_1)a_1 - 2\theta(d, s_2)a_2 + 2\bar{s}_1 a_0 a_1 + 2\bar{s}_2 a_0 a_2 \\ &\quad + 2\theta(s_1, s_2)a_1 a_2 + \theta(d, d) \end{aligned} \quad (11)$$

The θ 's are spatial correlations. $\theta(s_1, s_2) = \overline{s_1 s_2}$, etc. Note that $\theta(s_j, s_j) = \overline{s_j s_j} = 1$.

Adjusting the a 's to minimize $\bar{\epsilon}^2$ is equivalent to searching a parabolic stochastic surface (having as many dimensions as there are a 's) for a minimum. How well this surface can be searched will be limited by sample size, i.e., by the number of patterns seen in the searching process.

* The probability of neuron error is minimized by the adaption procedure subject to the restriction that $\bar{\epsilon} = 0$. This does not preclude the possibility that the error probability could be even less with neuron adjustments that will not cause $\bar{\epsilon}$ to be zero.

The method of searching that has proven most useful is the method of steepest descent. Vector adjustment changes are made in the direction of the gradient. The change made in a_0 is proportional to the partial derivative of c^2 with respect to a_0 , etc. The partial derivatives are determined at one point, then the changes in all adjustments are made simultaneously. This completes one iterative cycle. The process is then repeated. Transients decay along each adjusting coordinate in relaxation toward the stationary point. They consist of sums of geometric sequence components (there are as many natural "frequencies" as the number of adjustments, as can be seen from generalization of the flow graph of Fig. 8 -- see Ref. 9). If the proportionality constant k between partial derivative and size of change is made sufficiently small, transients will be stable. Just how big this constant could be for stable searching depends upon the surface characteristics (i.e., upon pattern characteristics). It can be shown, however, that when all second partial derivatives are equal (differentiation of Eq. 11 shows them all to have the value 2), the method of steepest descent will be stable when the proportionality constant k is less than the reciprocal of the second partial derivative. It can also be shown that when k is small, transients can be well represented as being of a single time constant. This time constant is somewhat sensitive to the specific pattern information, but generally turns out to equal $1/2k$.

When partial derivatives are measured by averaging over only a few patterns each iteration cycle, the measurements will be noisy, and transients will be noisy exponentials. Stability and time constant will remain dependent on k and the properties of the large-sample-size mean-square-error surface.

The method of adaption that has been used requires an extremely small sample size per iteration cycle, namely one pattern. One-pattern-at-a-time adaption has the advantages that derivatives are extremely easy to measure and that no storage is required within the adaptive machinery except for the gain values (which contain the past experience of the neuron).

The square of the error for a single pattern (the mean square error for a sample size of one) is given by Eq. (10). The partial derivatives are

$$\frac{\partial \epsilon^2(k)}{\partial a_0} = [-2d(k) + 2a_0 + 2s_1(k)a_1 + 2s_2(k)a_2]$$

$$\frac{\partial \epsilon^2(k)}{\partial a_1} = s_1(k)[-2d(k) + 2a_0 + 2s_1(k)a_1 + 2s_2(k)a_2]$$

$$\frac{\partial \epsilon^2(k)}{\partial a_2} = s_2(k)[-2d(k) + 2a_0 + 2s_1(k)a_1 + 2s_2(k)a_2] \quad (12)$$

Comparison of the Eqs. (12) with Eq. (9) shows that the derivatives are simply related to the measured error, and suggest that the derivatives could be measured without squaring and averaging and without actual differentiation. The j th partial derivative is given by

$$\frac{\partial \epsilon^2(k)}{\partial a_j} = -2s_j(k) \epsilon(k) \quad (13)$$

It follows that all derivatives have the same magnitude, and have signs determined by the error sign and the respective input signal signs. Application of the method of steepest descent requires that all gain changes in a given iteration cycle have the same magnitude and the appropriate sign. Each gain change reduces the error magnitude by the same amount. The procedure described in Sec. C for bringing $\epsilon(k)$ to zero with each successive input pattern gives the constant k a value of $1/(2(n+1))$. From the previous discussion we see that the time constant of the iterative process is therefore $\tau = (n + 1)$ patterns. On the 4×4 Adaline, there are $n = 16$ input line gains plus a level control. Therefore, the time constant should be roughly 17 patterns (for verification, see the learning curve of Fig. 5). The search procedure could be readily modified to speed up or slow down the adaption process. For example, bringing the error $\epsilon(k)$ to half its value rather than to zero with each input pattern halves k and doubles τ .

The statistical theory of adaption for sampled-data systems is based on search of multidimensional stochastic parabolic surfaces for stationary points. The misadjustment, a dimensionless measure of how well a system will adapt, is defined as the ratio of the mean increase in mean square error (due to searching the surface with small-sample-size data) to the

minimum mean square error (a performance reference that could only be achieved with perfect knowledge of input process statistics). The misadjustment Formulas (7) and (8) apply directly to the adaptive neuron.

The misadjustment formulas give the per unit increase in measured mean square error as a result of adapting on a finite number of patterns. Since the ratio of probability of neuron error to the mean square error $\overline{\epsilon^2}$ is essentially constant over a wide range of error probabilities (Fig.9), the misadjustment as expressed by Formulas (7) and (8) may be interpreted in terms of the ratio of the increase in error probability to the minimum error probability.

If adaption is accomplished by injection of a fresh pattern each iteration cycle, the mean values of the gains will converge, after adapting transients have died out, on the best set of values for large sample size. The actual gain settings will experience random excursions about these values, and the resulting misadjustment, as derived from Eq. (7) is

$$M = \frac{(n + 1)}{2\tau} \quad (14)$$

Following the procedure of bringing $\epsilon(k)$ to zero each iteration cycle, the misadjustment is

$$M = \frac{(n + 1)}{2\tau} = \frac{(n + 1)}{2(n + 1)} = \frac{1}{2} \quad (15)$$

If adaption is accomplished by taking a fixed collection of N patterns and repeating them over and over for several time constants (where the time constant is long, several times N), the gains will stabilize on the best set of values for the N patterns. In general, these gains will not be the best for the large collection of patterns that the N patterns were abstracted from. Making use of Eq. (8), the misadjustment is

$$M = \frac{(n + 1)}{N} \quad (16)$$

An extensive series of simulation studies has been made to test the validity of the misadjustment Formulas (14) and (15). These tests have shown that the formulas are highly accurate over a very wide range of pattern and noise characteristics. A description of a typical experiment and its results is given in Fig. 10.

$X \rightarrow +1$

$T \rightarrow -1$

10% Noise

$C \rightarrow +1$

$J \rightarrow -1$

Best neuron makes 12 errors out of 100

EXPERIMENT #	PATTERNS ADAPTED ON	NUMBER OF ERRORS	MISADJUSTMENT
1	95, 79, 07, 60 73, 61, 08, 02, 72, 26	25	$M = \frac{25-12}{12} = 108\%$
2	70, 69, 52, 55, 32 97, 30, 38, 87, 01	19	$M = \frac{19-12}{12} = 58\%$
3	65, 12, 84, 83, 34 38, 71, 66, 13, 80	20	$M = \frac{20-12}{12} = 67\%$
4	07, 42, 85, 88, 63 35, 37, 92, 79, 22	28	$M = \frac{28-12}{12} = 133\%$

FIG. 10.--EXPERIMENTAL ADAPTION ON 10 NOISY 3x3 PATTERNS.

Noisy 3×3 patterns were generated by randomly injecting errors in ten percent of the positions of the four "pure" patterns, X, T, C, J. These patterns, shown in Fig. 10, are ordered for convenience in checking. They were fed manually to Adaline and chosen randomly by looking up their identification numbers in a random number table. The X's (numbered from left to right, up to down) were numbered 1 to 25, the T's were 25 to 50, the C's were 50 to 75, and the J's were 75 to 100.

The best system was arrived at by slow precise adaption on the full body of 100 noisy patterns, repeating them over and over several times. This system was able to classify the patterns as desired, except for twelve errors out of the 100 total. The gains were then set to zero and ten patterns were chosen at random. The best system for the ten selected patterns was arrived at by slow adaption on these patterns, repeating them over and over several times. The resulting system was then tested on the full body of 100 patterns, and 25 classification errors out of 100 were made. This number of errors was more than twice that made by the best system adapted on 100 patterns. The misadjustment was 108 per cent. This small-sample-size adaptation experiment was repeated three more times, and the misadjustments that resulted, in order, were 58 per cent, 67 per cent and 133 per cent. Since $N = 10$ patterns and $n = 9$ input lines, the theoretical misadjustment was

$$M = \frac{n + 1}{N} = 100 \text{ per cent}$$

An average taken over the four experiments gives a measured misadjustment of 91.5 per cent.

The adaptive classifier can adapt after seeing remarkably few patterns. A misadjustment of 20 per cent should be acceptable in most applications. To achieve this, all one has to do is supply the adaptive classifier with a number of patterns equal to five times the number of input lines, regardless of how noisy the patterns are and how difficult the "pure" patterns are to separate. Although the misadjustment formulas have been derived for the specific classifier consisting of a single adaptive neuron, it is suspected that the following "rule of thumb" will apply fairly well to all adaptive classifiers: the number of patterns required to train an adaptive classifier is equal to several times the number of bits per pattern.

output. The read-out oscillator provides the a-c current to operate the read-out wires. The read-out amplifier converts the a-c output signals to a d-c output signal. The summer (Σ), computes the error $\epsilon(k)$. The error sign circuit computes $\text{sgn } \epsilon(k)$. The error magnitude circuit provides a signal which blocks the "and" gate when the magnitude of the error falls below a preset level, thus preventing the operation of the adapt-drive circuit. Therefore, when the "Adapt" signal is applied, the adapt-drive circuit is repeatedly energized until the error falls below the preset level. The delay circuit controls the amount of time between energizations of the adapt-drive circuit. This time must be long enough to allow the error to reach its new value after each energization.

When networks of neurons are used, it is possible that a single set of driving circuits could be employed to actuate all of the adaptive neurons. At present, this is not practical for large networks because of the power levels required. The MAD elements shown in Fig. 14 are quite large, and might ultimately be able to be made much smaller, perhaps in the form of thin films. It should ultimately be possible to mass produce large networks of adaptive microelectronic logical elements. Power levels should be low, space and weight requirements and cost should be low. These neurons should be thought of and treated as new kinds of circuit elements, adaptive logical components.

VIII. APPLICATIONS FOR ADAPTIVE LOGICAL CIRCUIT ELEMENTS.

The field of application of digital systems may be classified into two broad categories, fixed systems and adaptive systems. The structure of the fixed system is completely determined by the designer, while the adaptive system is designed to have both fixed and adjustable portions. The latter system has the ability to automatically modify its adjustable parts by trial and error experience in order to optimize performance (this is performance feedback). Fixed systems are by far the most common at present. Adaptive systems have received intensive study during the past several years, and some practical applications are being made in automatic control and in the recognition (classification) of pattern information.

Both sets of patterns were fed to two Adalines simultaneously and perfect adaption was possible. The adaption procedure was the following: if the desired output for a given input pattern applied to both machines was -1, then both machines were adapted in the usual manner to ensure this; if the desired output was +1, the machine with the smallest measured error ϵ was assigned to adapt to give a +1 output while the other machine remained unchanged. If either or both machines gave outputs of +1, the pattern was classified as +1. If both machines gave -1 outputs, the pattern was classified as -1.

This procedure assigns specific "responsibility" to the neuron that can most easily assume it. If, at the beginning of adaption, a given neuron takes responsibility for producing a +1 with a certain input pattern, it will invariably take this responsibility each time the pattern is applied during training. Notice that it is not necessary for a teacher to assign responsibility. The combination does this automatically and requires only input patterns and the associated desired outputs, like the single neuron.

More complicated problems can be well solved by combinations of many neurons. Their inputs are connected in parallel while their outputs are connected to an OR element. The only new requirement is that of the job assigner, which is simple to implement. Such combinations greatly increase the generality of the classification scheme, and the ease of adaption is comparable to that of a single neuron. A theory of adaption for these combinations has yet to be completed. Preliminary considerations indicate that the misadjustment formulas will apply without appreciable change when combinations of neurons adapt on noisy nonlinearly separable patterns.

Various classification problems could be solved simultaneously by multiplexing neurons or combinations of neurons. One neuron might be trained to decide whether the man in a given picture does or does not have a green tie, while another neuron or combination could be trained to decide whether or not the man has a checkered shirt. Each neuron or combination has its own output line, and each is fed the appropriate desired output signal during training. The input signals are common to all neurons. In this manner, it is possible to form adaptive classifiers that can separate with great accuracy large quantities of complicated patterns into many output categories. All that is needed is large quantities of adaptive neurons.

VII. ADAPTIVE MICROMECHANICAL SYSTEMS

The structure of the neuron described in this report and its adaption procedure is sufficiently simple that an effort is under way to develop a physical device which is an all-electronic fully automatic Adaline. The objective is a self-contained device, like the one sketched in Fig. 11, that has a signal input line, a "desired output" input line (actuated during training only), an output line, and a power supply. The device itself should be suitable for mass production, should contain few parts, should be reliable, and probably should consist of solid-state components.

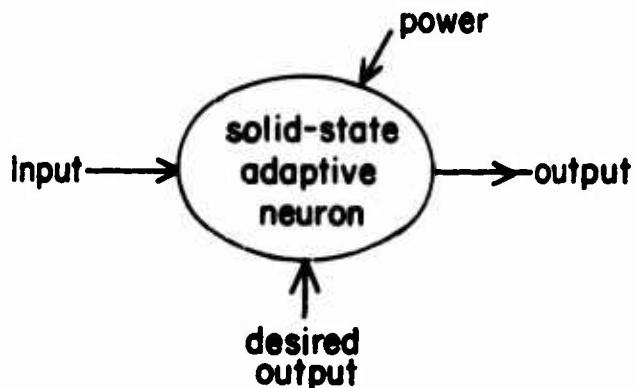


FIG. 11...ELECTRONIC AUTOMATICALLY-ADATED NEURON.

To have such an adaptive neuron, it is necessary to be able to store the gain values, which could be positive or negative, in such manner that these values could be changed electronically.

Present efforts have been based on the use of multi-aperture magnetic cores (MAD elements¹¹). The special characteristics of these cores permit multilevel storage with continuous, non-destructive read out. In addition, the stored levels are easily changed by small controlled amounts, with the direction of the change being determined by logic performed by the MAD element.

Figure 12 shows a block diagram for an electronic adaptive element, which realizes the adaption technique described previously. The MAD element array contains magnetic cores and wire only; Fig. 13 shows how

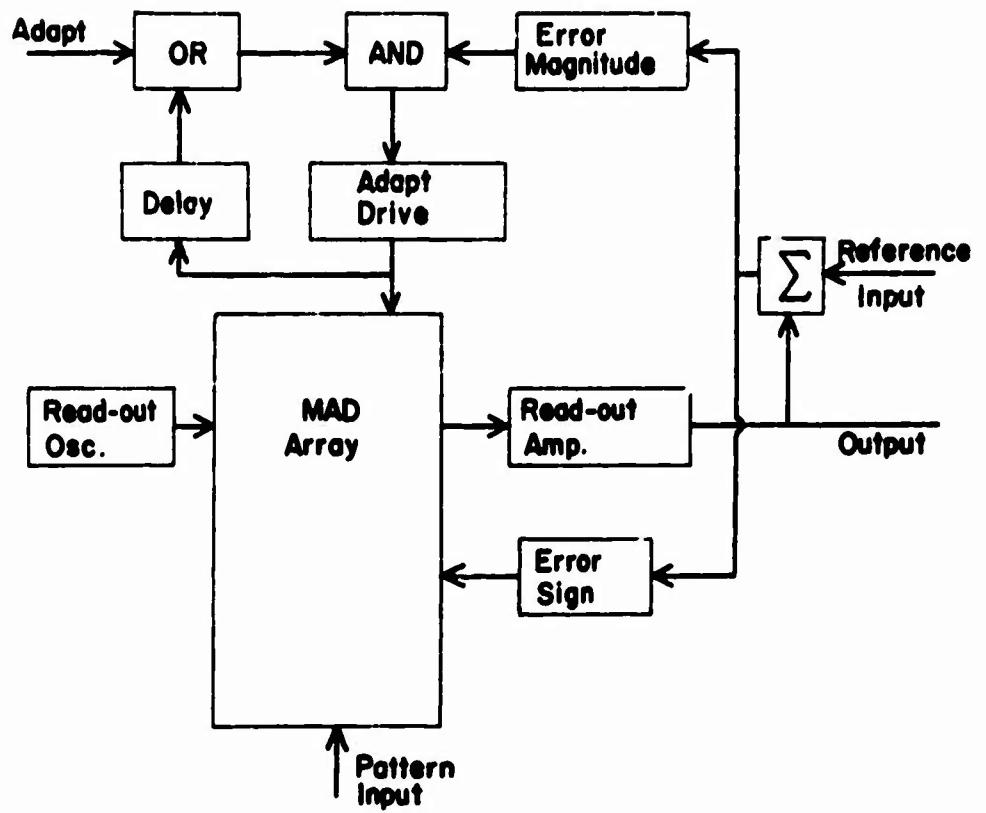


FIG. 12.--BLOCK DIAGRAM, ELECTRONIC ADAPTIVE ELEMENT.

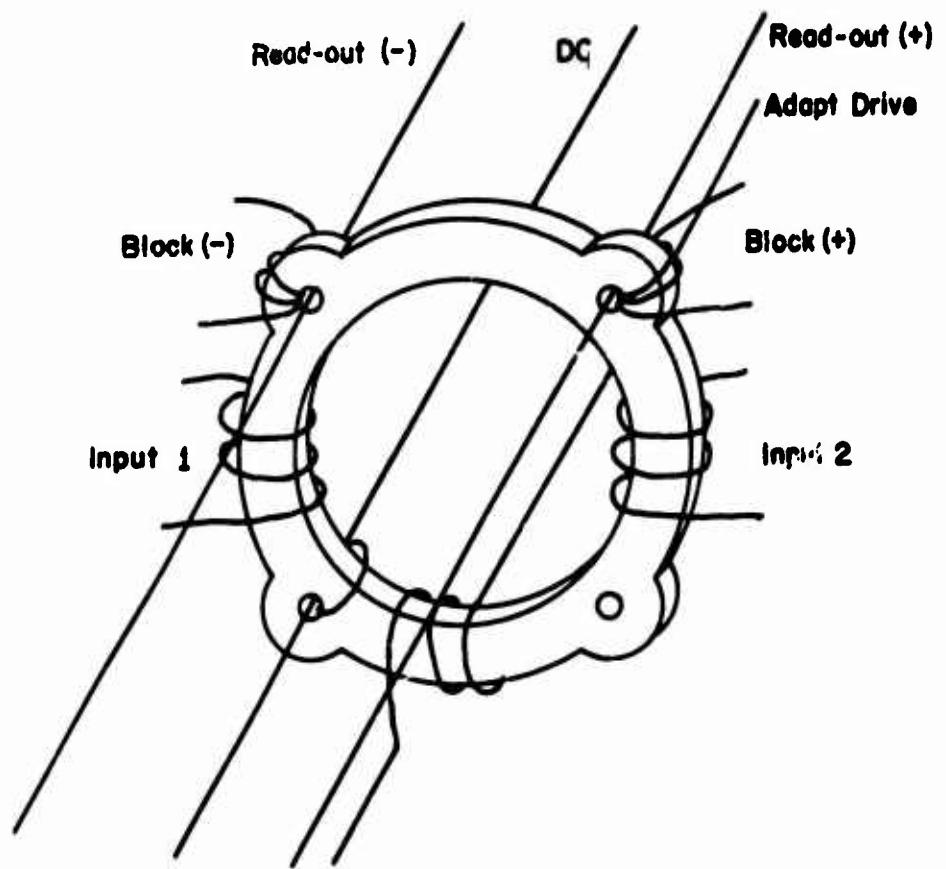


FIG. 13.--MAD ELEMENT—WINDINGS FOR USE IN ELECTRONIC ADAPTIVE ELEMENTS.

each core is wired. A photograph of the first experimental array for a 5x5 Adaline is shown in Fig. 14. This array performs the following functions:

1. Storage of the gains and the d-c level (the a_i 's). This storage is passive, i.e., the information is not lost in the event of power failure. There is one MAD element for each gain and one for the d-c level. Thus for $m \times n$ patterns $mn + 1$ MAD elements are required.
2. Continuous computation of the sum $a_0 + \sum_i a_i s_i(k)$ for the pattern connected to the input. The sum appears as two a-c signals, one appearing across each of the read-out wires. The signal across one of these wires corresponds to the sum of those terms for which the $s_i(k)$ is negative; the other corresponds to the sum of the d-c level and those terms for which the $s_i(k)$ is positive. [Each read-out wire carries an a-c current. The voltage drop per core on a read-out wire is a linear function of the value of the gain stored in that core, provided that the aperture through which the wire passes is not blocked by energizing the block winding of that aperture. If blocked, the voltage drop is very small. Thus, the summation is accomplished by energizing the "Block (-)" winding of the i^{th} core when $s_i(k)$ is negative and energizing the "Block (+)" winding when $s_i(k)$ is positive.]
3. Computation of the adaption change δa_i in the gain a_i . Each of these changes is proportional to the product $s_i(k) \text{sgn}[\epsilon(k)]$. [The change in the stored level of the core is accomplished by applying the proper signal to the "Adapt Drive" wire. With the proper adapt-drive waveform, the direction of the change may be reversed by applying a d-c bias to one of the "Input" windings. Input 1 is energized when both $s_i(k)$ and $\epsilon(k)$ are positive; Input 2 is energized when both are negative. For $s_i(k)$ and $\epsilon(k)$ of opposite sign, no current is applied to either input.] All of the changes δa_i are of the same magnitude. To reduce the error to approximately zero, the Adapt Drive wire is energized a sufficient number of times. The d-c winding on the MAD element carries a d-c bias current. This current may be removed between adapt-drive signals, but must be applied during the adapt-drive signal.

The peripheral circuitry supplies the necessary signals to the MAD element array, and converts the a-c read-out signals to a more useful d-c

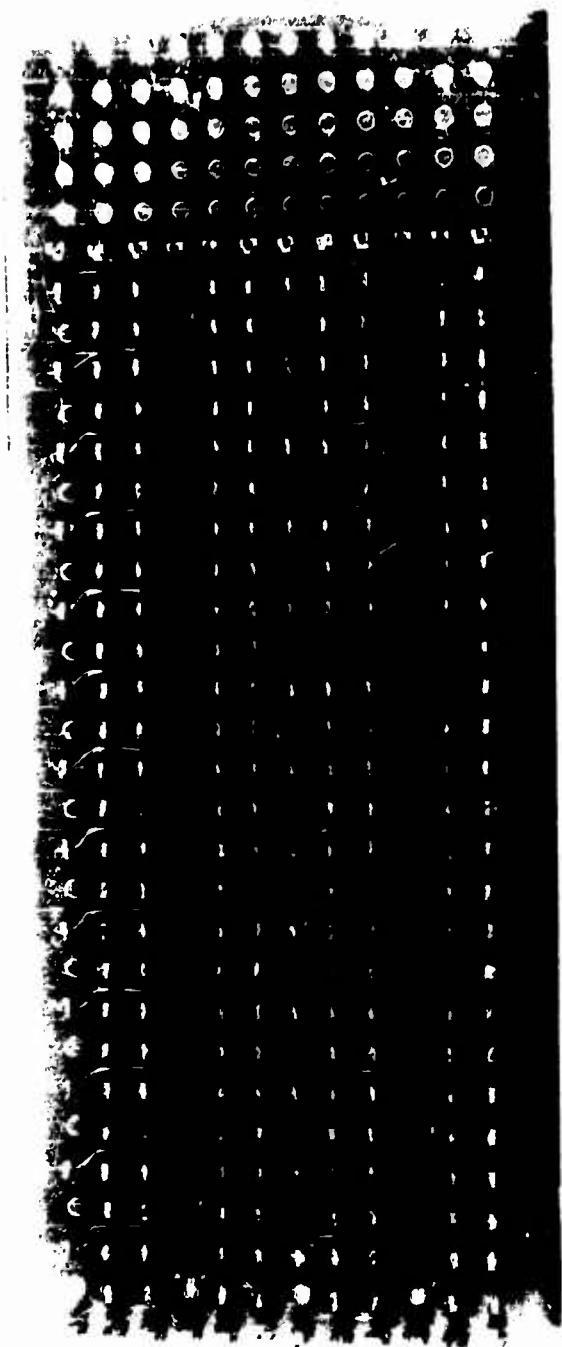


FIG. 14.--EXPERIMENTAL AUTOMATIC ADALINE.

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Both fixed and adaptive systems may be realized by programming general purpose digital computers. These computers on the other hand are realized of conventional logical components (flip-flops, gates, etc.), but may be realized of networks of adaptive neurons. All details of organization, design, and construction of computers must be completely planned in the present day scheme of things. If a computer were built of adaptive neurons, details of structure could be imparted by the designer by training (showing it examples of what he would like it to do) rather than by direct designing. This design concept becomes more significant as size and complexity of digital systems increase. The demands of modern technology are such that larger and more complex digital systems are continually being contemplated, and in step with this, progress in microelectronics makes such systems physically and economically possible.

The problem of reliability is greatly aggravated by increase in size and complexity. Significant steps in improving the reliability of digital systems have been made, notably with the introduction of the magnetic-core memory, and the use of high-speed switching transistors as active logical elements. Although the reliability of individual components has constantly increased, the requirement in numbers of components has increased in many cases far more rapidly. It is not expected that mass-produced microminiature components will ever be perfectly reliable, yet they will be usable in large systems. The problem is to devise new systems techniques to achieve reliable over-all operation where systems consist of large numbers of interacting imperfect components.

Errors caused by computer component failure are, in general, more deleterious to a fixed system. In the event of a failure, the adaptive system will adjust whatever remains adjustable to do the "best" job with the intact parts. As long as the adaption mechanism is reliable, system reliability is inherently increased.

Shannon and Moore¹² and von Neumann³ have proposed schemes for making reliable fixed digital systems from unreliable components by using redundancy. Another method, using adaptive logic, is hereby proposed for improving system reliability.

The reliability of a system whose purpose is non-adaptive may be increased by combining adaption and redundancy. Consider a multiplex

consisting of three machines solving the same problem with the same input data. Let the output of each machine be a single binary number, expressed as +1 or -1. If these machines were perfectly reliable, their outputs would always agree. If not, then von Neumann proposed that the majority should rule. The neuron shown in Fig. 1 with a_0 set to zero, and the other gains set to +1 would give a majority output. Each machine has equal vote. Unequal vote (higher vote going to the more reliable machine) is possible by making the a 's adjustable, and causing these adjustments to be made automatically to optimize performance. The adaptive vote taker is identical to the adaptive neuron. The vote taker can be trained by periodically injecting a certain input when the desired output is known.

von Neumann's majority rule vote taker will give the correct outcome when the majority is correct. The adaptive vote taker could ideally give the correct outcome with only a single correct machine by giving it a heavy vote and attenuating the votes of the unreliable machines. This is in effect an adaptive routing procedure for information flow, and allows systems in a small measure to be self-healing.

The effectiveness of the adaptive vote taker is being evaluated by William Pierce in a doctoral thesis research at Stanford University. It has been shown that the effective multiplex factor can be greatly increased by adaption (particularly where the machines are fairly unreliable), and that system life expectancy can also be greatly increased by adaption and redundancy. This work will be described in a Stanford University technical report.

When adaptive neuron elements become available in large quantities, adaptive logical and computing systems will probably be organized quite differently from the way modern computing systems are organized. The organizations of two related adaptive system types will be considered, that of adaptive pattern classifiers and of adaptive problem-solving machines.

The realization schemes utilized by Clark and Farley,¹³ Rosenblatt,¹⁴ and Mattson^{5,6} for adaptive pattern classifiers made use of digital simulation. The approach suggested by this work is that adaptive pattern classifiers be constructed of networks of adaptive neuron elements.

One of the most promising areas of research in computer system theory is that of problem-solving machines, theorem-proving machines, and artificially "intelligent" machines. The earliest proponents of this research were Turing¹⁵ and Shannon.¹⁶ Their suggestions were successfully put to practice by Newell, Simon, and Shaw,¹⁷ by Samuel,¹⁸ and by others. Problem-solving has been regarded as a multistage decision process which begins with an initial status and ends with a goal status. Each change in status results from the selection of a certain move from a collection of possibilities which are "legal" according to the rules of the game. Since the number of chains of moves increases approximately exponentially with the length of the chains, exhaustively trying all chains in search of a goal is not practical, even for simple problems.

The approach taken by Samuel in his checker-playing simulations to reduce the number of chains to be tested was two-fold. The length of the chains was limited to be somewhere between ten and thirty moves ahead (a "ply" of 10 to 30), and since most chains would not terminate by reaching goals, a system of status evaluation was developed so that the various chains could be numerically compared. The second method of reducing the number of chains to be tested was to check against games stored in the memory. If an identical situation was encountered previously, certain evaluations have already been made and need not be repeated. This use of stored games was called "rote learning". A procedure for making the status evaluation system adaptive was called "generalized learning". Both of these learning methods could be used simultaneously.

The rote learning portion of the over-all procedure could be made to be much more powerful if it were possible to extract from the memory previous situations that are similar (are not necessarily identical) to the current situation. Far less experience and storage would be needed to reach a given level of competence of play. Similar means that the previous situation is in the same subclass with the current situation. A classification scheme would be needed to establish similarities in checker situations. The structure of this classifier would have to be formed from experience.

An automatic problem-solving computer should have a memory system from which information could be extracted according to classification

rather than by address number. The extent of classification before storing should be slight (e.g., is the pattern of checkers or of chess?), and a consistent scheme for the arrangement of the pattern bits should be established before storing. Final classification should be done within the memory itself. Each storage register should contain an Adaline or a network of Adalines.

A request from a "central control" for a certain type of information is sent to every register in the memory simultaneously. This has the effect of setting the adjustments of all the Adalines. Only the registers whose classifiers respond properly (e.g., give +1 outputs) answer the request and transmit their information back to the "central control".

Very sophisticated learning procedures would become possible if one has such recall-by-association parallel-access memory systems. The simplicity of Adaline and the progress being made in microelectronics gives a strong indication that such memory systems will come into existence in the not too distant future.

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